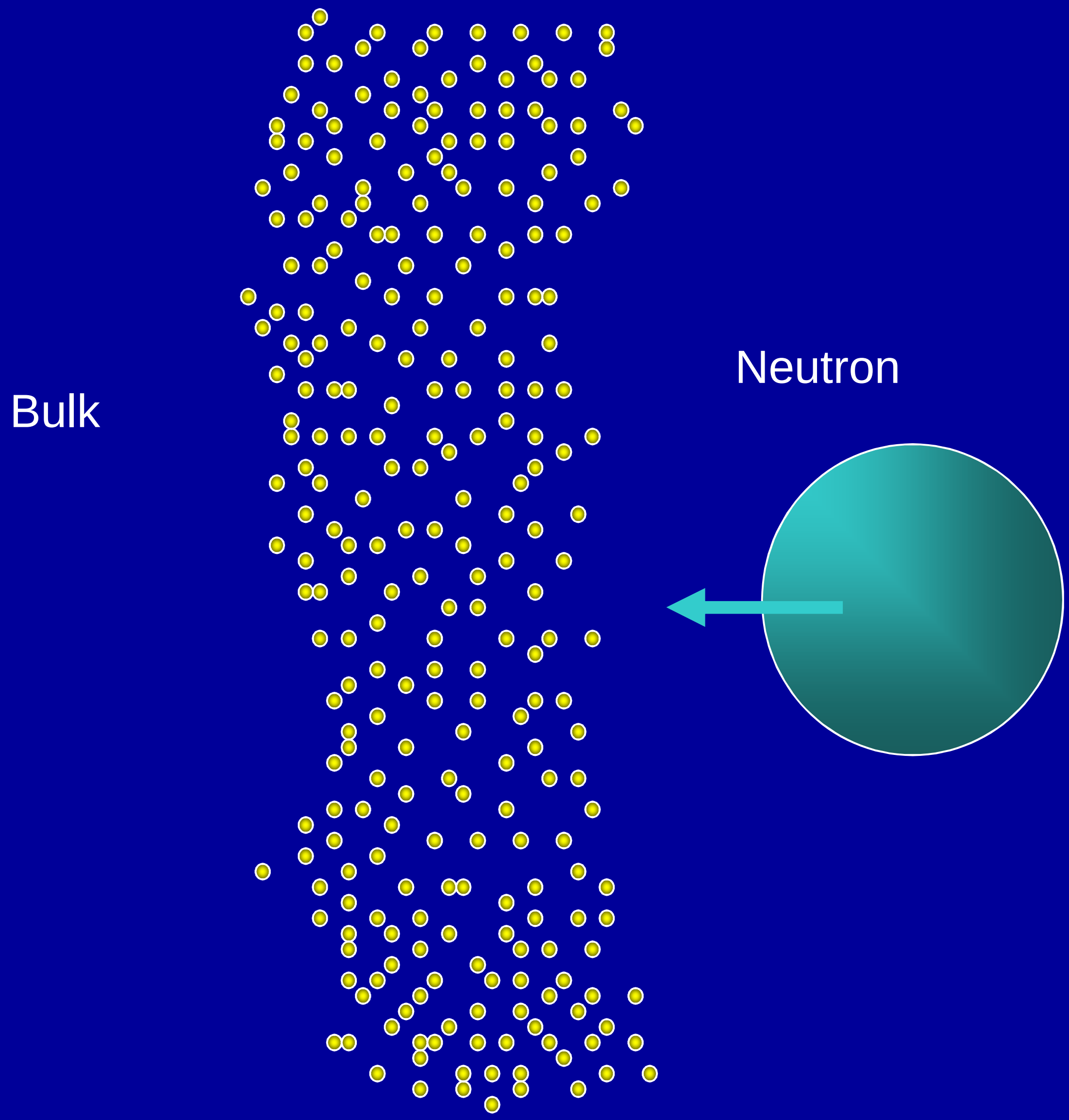
Properties of the Neutron

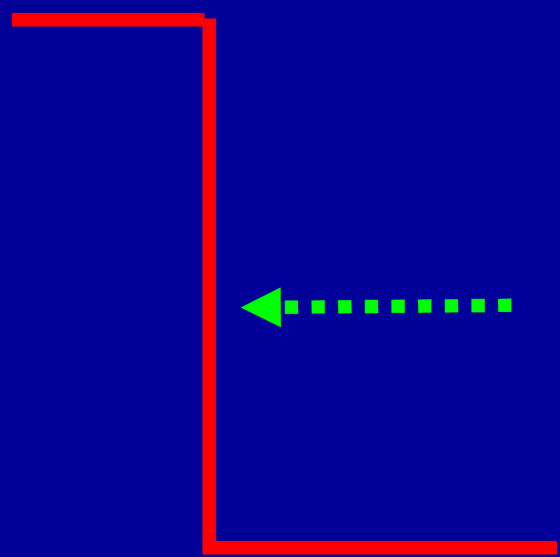
- Properties
  - Ultra Cold Neutrons
- “Whispering” gallery phenomenon
  - Motivations
  - Scattering on a cylinder
  - Perspectives

Fermi Potential



Fermi potential seen by neutron

$$U = \frac{2\pi\hbar^2}{m} N b_{coh}$$



Properties of Ultra Cold Neutrons

Interaction with matter

~ 99.99 % – elastic reflection

~ 10^{-4} – inelastic diffusion on thermal phonons

~ 10^{-5} – absorption

$V_{UCN} \sim (1 \div 10)$ m/s

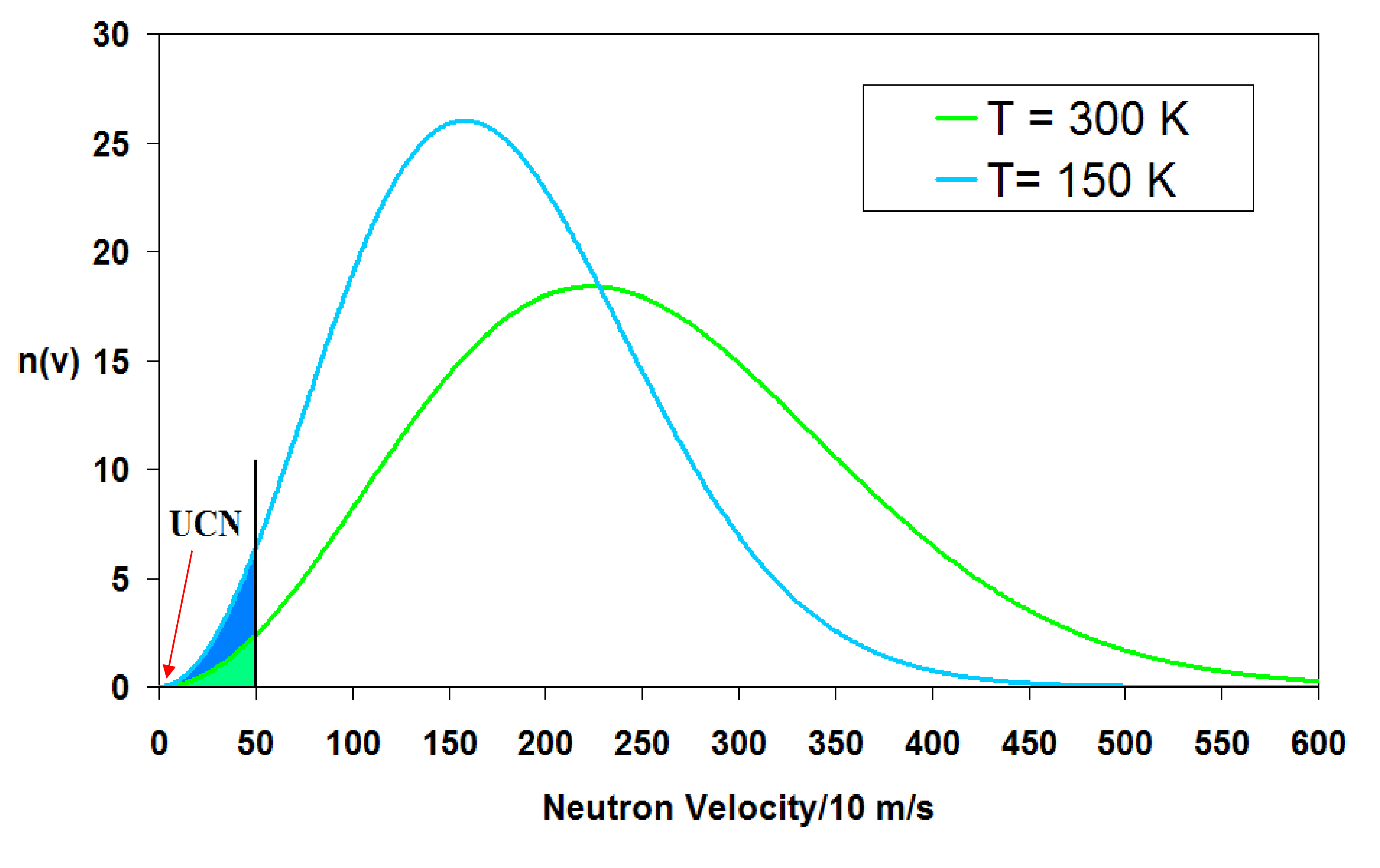
$E_{UCN} \sim 1$ neV

$H_{UCN}^{Earth's\ field} \sim 1$ m

$T \sim 1$ mK ← name UCN

Material	b_{coh}, fm	Density, g/cm^3	V_{lim}, m/s
D_2 (liquide)	13	0,15	3,82
D_2O	18,8	1,1	5,57
C (graphite)	6,65	2,25	6,11
C (diamant)	6,65	3,52	7,65
Al_2O_3	24,2	3,7	5,13
SiO_2	15,8	2,3	4,26
Acier	8,6	8,03	6,0

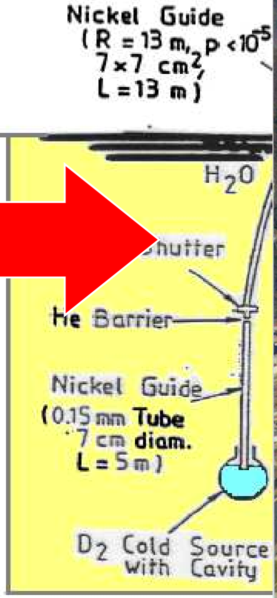
Necessity of cooling



ILL UCN source (guide and turbine)

ULTRACOLD NEUTRON FACILITY

ILL PF2



At the experiment the
UCN Number density for v < 6 m/s

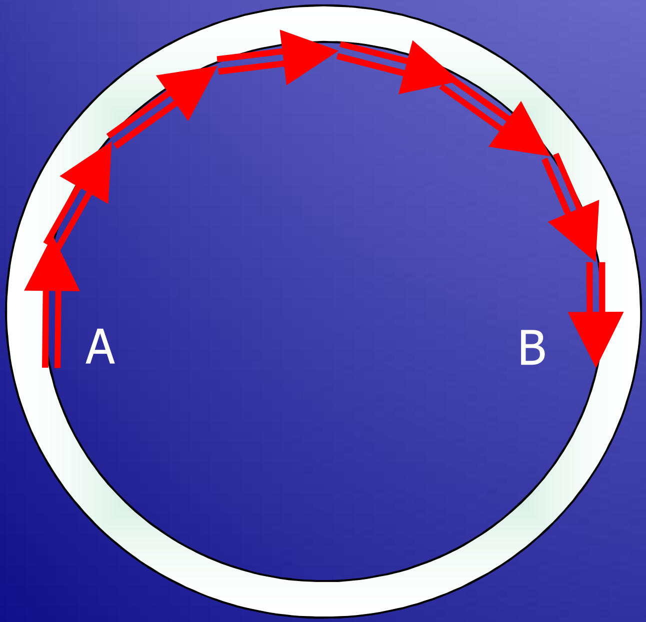
50 /cm3 or

50000 /litre

« Centrifugal » states

as a realization of the « whispering gallery » for particles

St Paul Cathedral : The circular gallery which runs at the point where the vault of the Dome starts to curve inwards, is called the Whispering Gallery. The name comes from the fact that a person who whispers facing the wall on one side, can be clearly heard on the other, since the sound is carried perfectly around the vast curve of the Dome

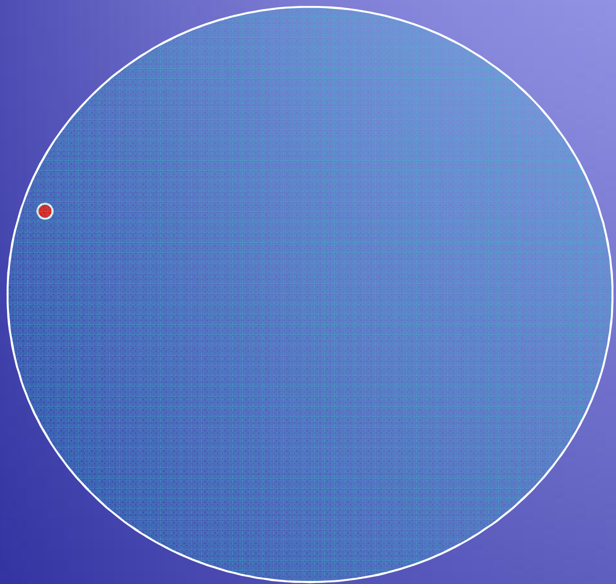


$$I(r_{AB}) \sim r_{AB}^{-1}$$

[1] J. W. Strutt Baron Rayleigh, *The Theory of Sound* (Macmillan, London 1878), Vol. 2.

[2] L. Rayleigh, Philos. Mag. 27, 100 (1914).

And if one captures a particle?



This particle will be confined between a wall and a centrifugal barrier

Quantum motion

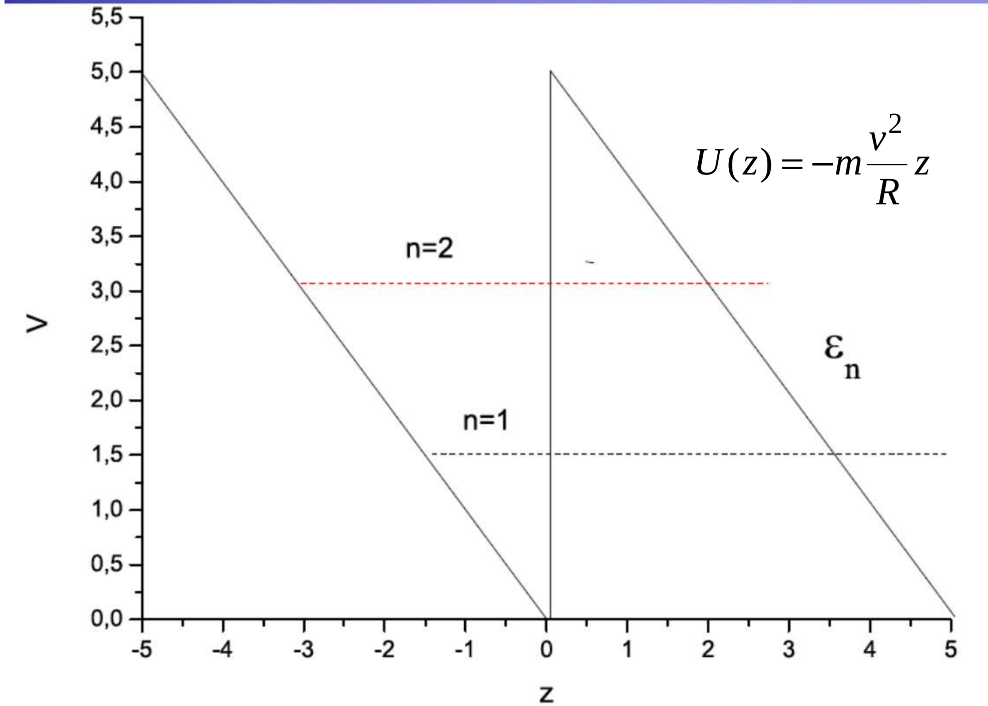
V.V. Nesvizhevsky, A. K. Petukhov, K. V. Protasov, A. Yu. Voronin
"Centrifugal quantum states of neutrons", Phys. Rev. A **78**, 033616 (2008)

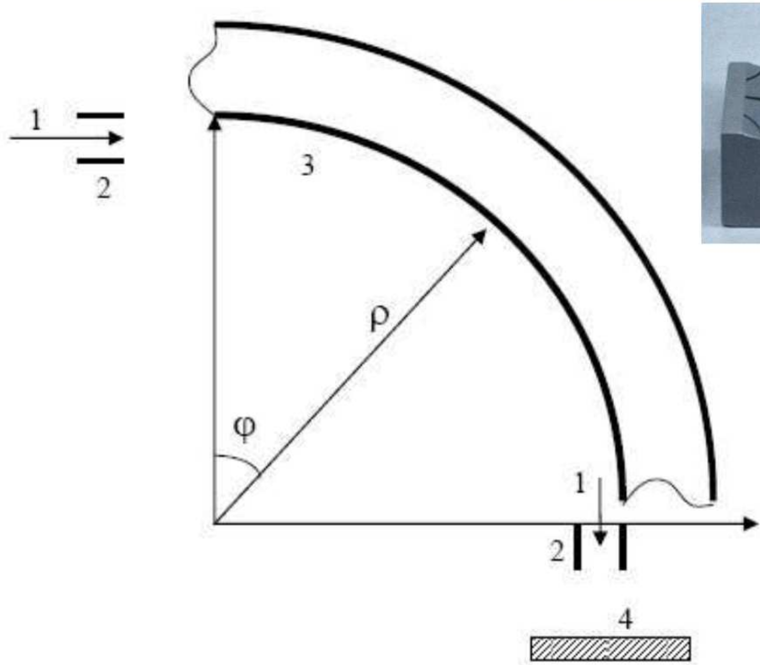
V.V. Nesvizhevsky, A. Yu. Voronin, R. Cubitt, K. V. Protasov,
"Neutron whispering gallery", Nature Physics, **6** (2010) 114

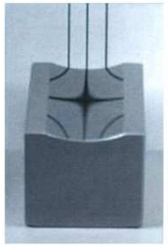
V.V. Nesvizhevsky, A. Yu. Voronin, R. Cubitt, K. V. Protasov,
"The whispering gallery effect in neutron scattering", New J. Phys., **12** (2010) 113050

Centrifugal states

These quasi stationary states decay by tunneling through the mirror





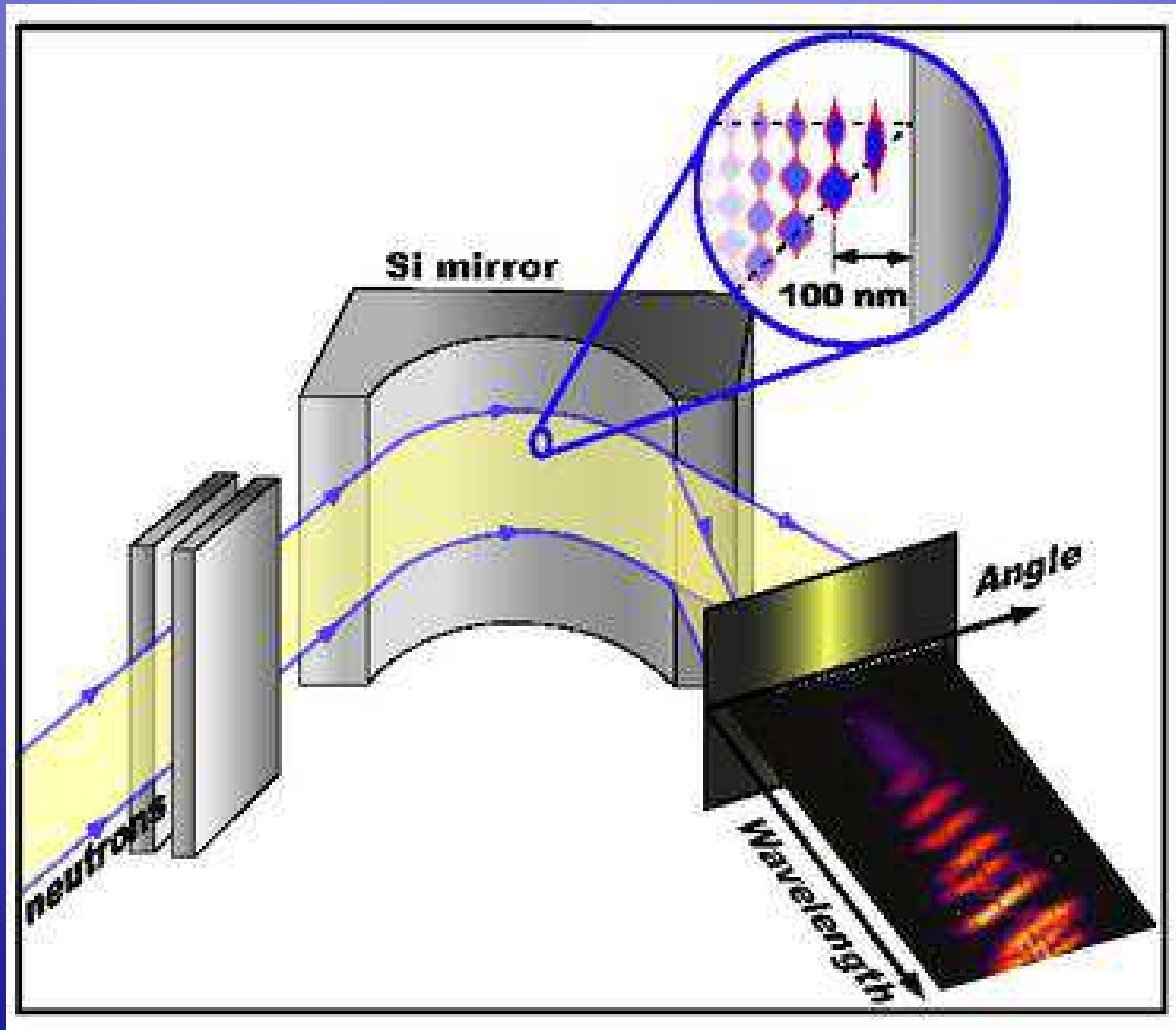


$$\Psi \propto \begin{cases} \mathrm{Bi}(-z/l_0 - \varepsilon_n/\varepsilon_0) + i\mathrm{Ai}(-z/l_0 - \varepsilon_n/\varepsilon_0) & z < 0 \\ \mathrm{Ai}(-u_0 - z/l_0 - \varepsilon_n/\varepsilon_0) & z \geq 0 \end{cases};$$

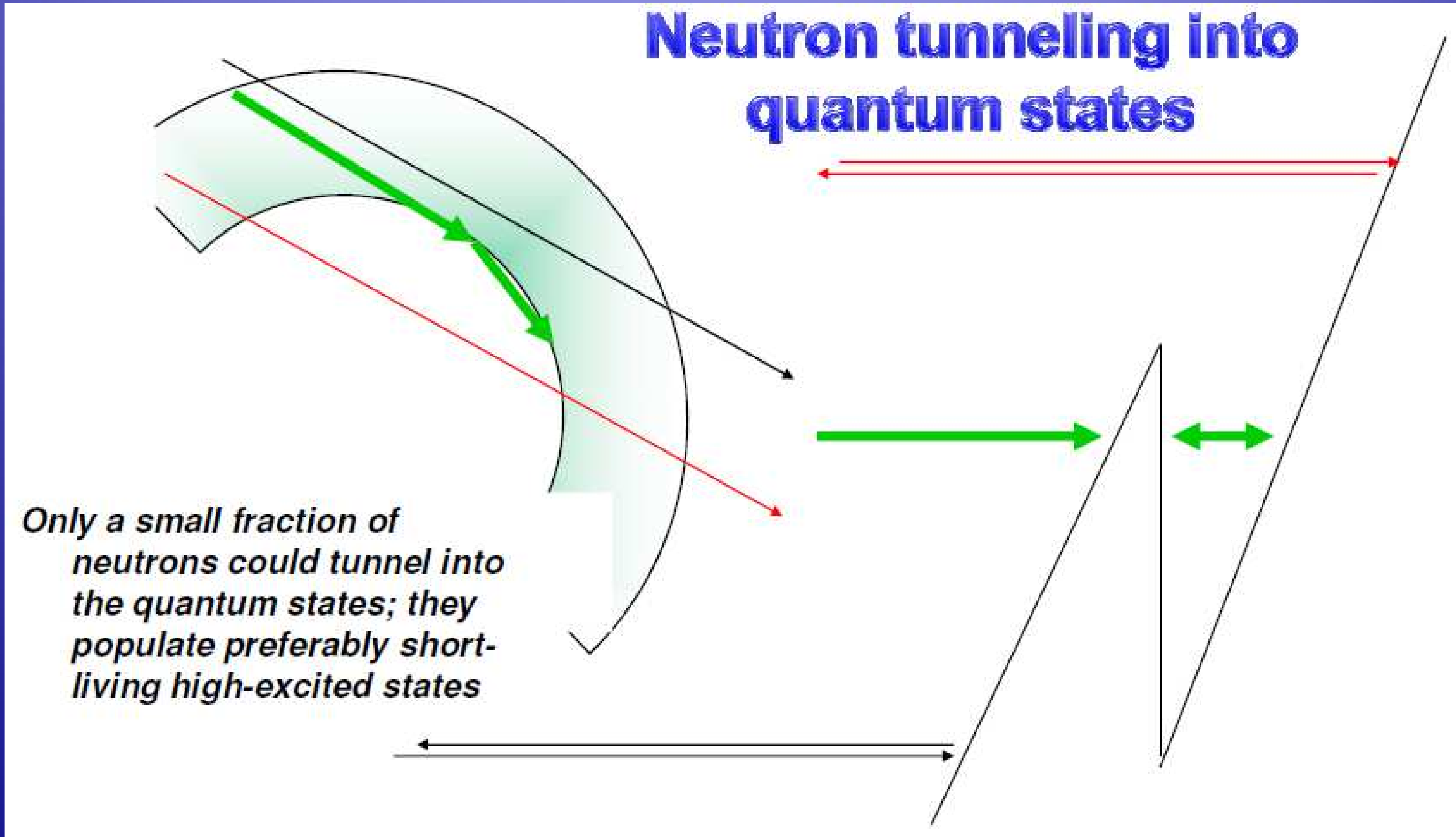
$$\varepsilon_0 = \left(\frac{\hbar^2 M v^4}{2R^2}\right)^{1/3} - \text{characteristic energy},\; l_0 = \left(\frac{\hbar^2 R}{2M^2 v^2}\right)^{1/3} - \text{characteristic length}$$

$$u_0 = V_F/\varepsilon_0,\; V_F - \text{ mirror bulk Fermi potential}$$

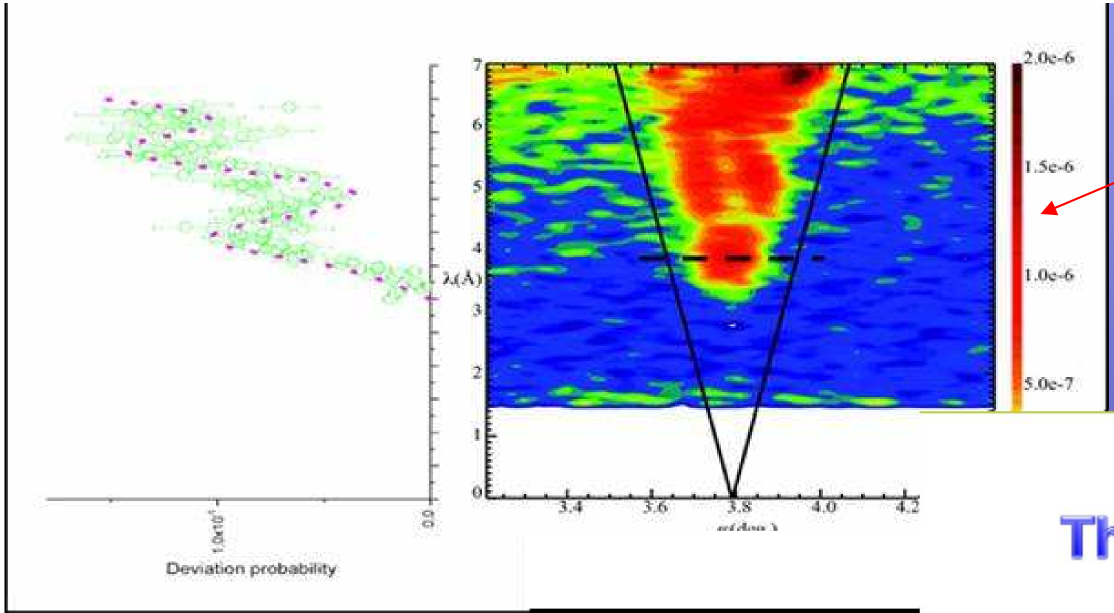
Scheme of the experiment



How to populate these states?

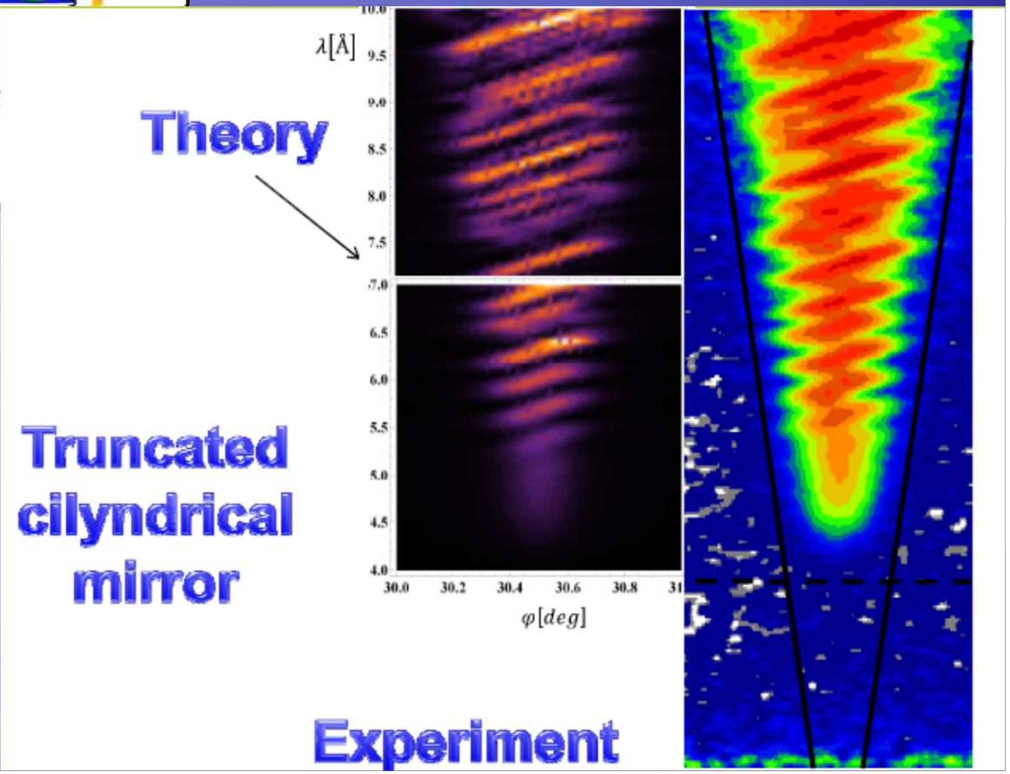


First results



States close to the top of the barrier

Deeply bound states



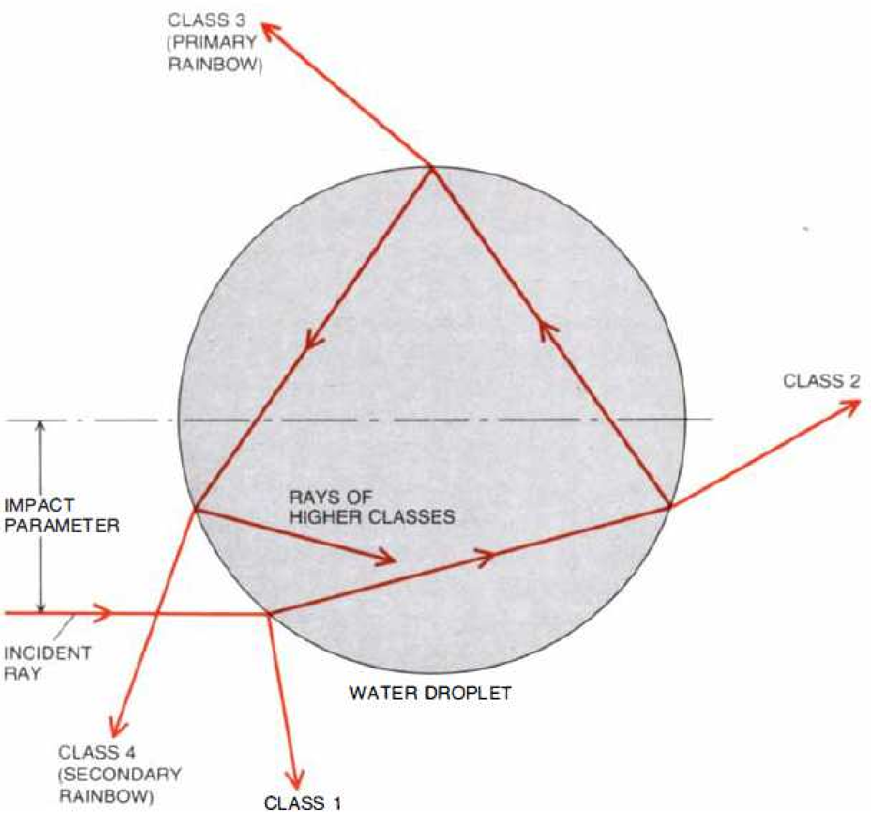
Quite complicate
(but extremely reach
and very informative) picture of
interference

How to analyze this quantum mechanical solution?

- The problem (neutron scattering by a cylinder) is quite similar to the scattering of light by a sphere (rainbow, gloria, surfaces waves,... phenomena)
- In the last case, the solution was proposed in 60th by H. Moysés Nussenzveig and is called Complex Angular Momentum approach [see J.A.Adam, “The mathematical physics of rainbows and glorias”, Phys. Reports, 356 (2002) 229]

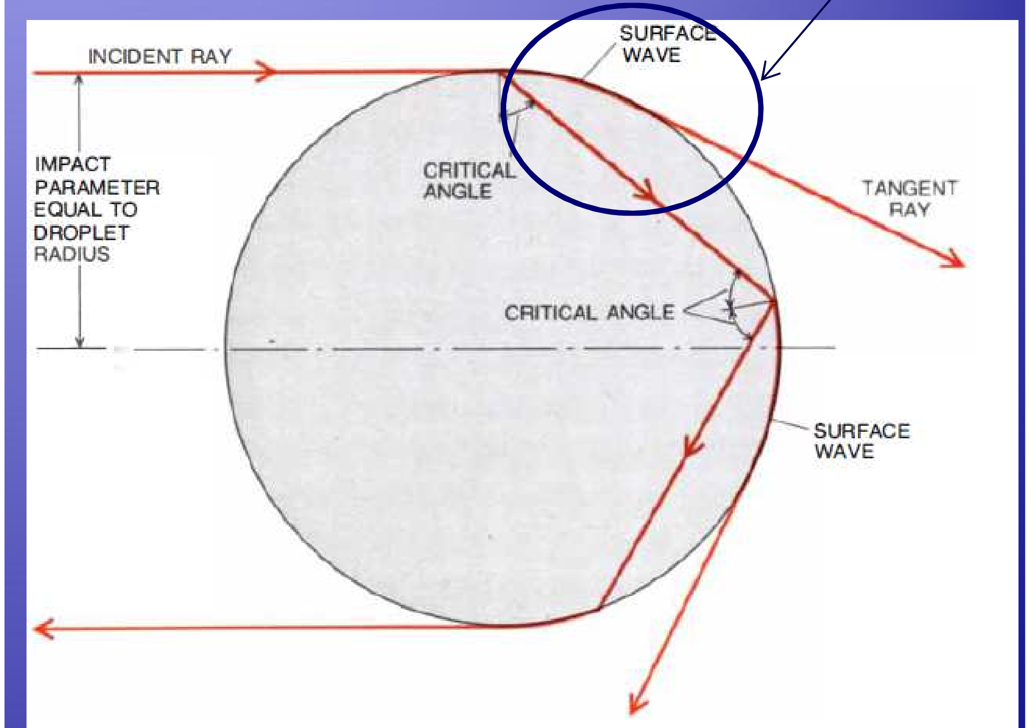
The Theory of the Rainbow

by H. Moysés Nussenzveig (Scientific American, 1977)



PATH OF LIGHT through a droplet can be determined by applying the laws of geometrical optics. Each time the beam strikes the surface part of the light is reflected and part is refracted. Rays reflected directly from the surface are labeled rays of Class 1; those transmitted directly through the droplet are designated Class 2. The Class 3 rays emerge after one internal reflection; it is these that give rise to the primary rainbow. The secondary bow is made up of Class 4 rays, which have undergone two internal reflections. For rays of each class only one factor determines the value of the scattering angle. That factor is the impact parameter: the displacement of the incident ray from an axis that passes through the center of the droplet.

Whispering gallery

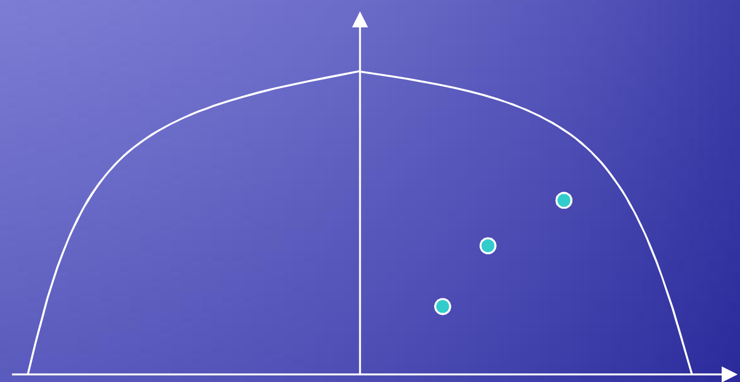


COMPLEX-ANGULAR-MOMENTUM theory of the rainbow begins with the observation that a photon, or quantum of light, incident on a droplet at some impact parameter (which cannot be exactly defined) carries angular momentum. In the theory, components of that angular momentum are extended to complex values, that is, values containing the square root of −1. The consequences of this procedure can be illustrated by the example of a ray striking a droplet tangentially. The ray stimulates surface waves, which travel around the droplet and continuously shed radiation. The ray can also penetrate the droplet at the critical angle for total internal reflection, emerging either to form another surface wave or to repeat the shortcut.

The sum is replaced by the integral ("Watson transformation")

$$f(k,\varphi) \rightarrow \frac{1}{i}\sqrt{\frac{\hbar}{2\pi k}}\int_{-\infty}^{+\infty}\left(S_m(k)-1\right)e^{im\varphi}dm + \dots$$

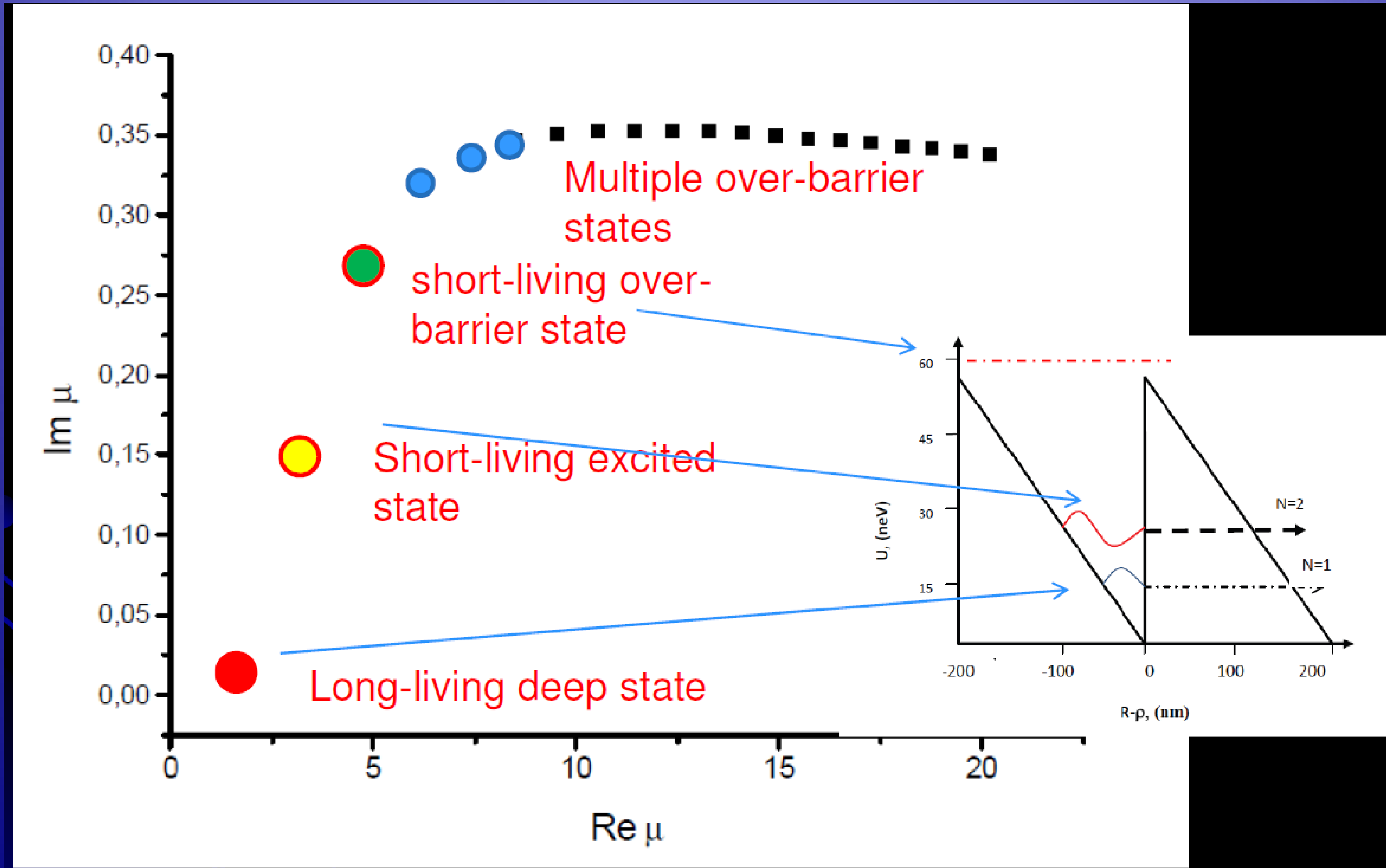
In the complex m -plane



$$f(k,\varphi) \rightarrow \sqrt{\frac{2\pi\hbar}{k}}\sum_i \mathrm{Res}\, S_{\mu_i}(k)\, e^{i\,\mathrm{Re}\,\mu_i\varphi}e^{-\mathrm{Im}\,\mu_i\varphi} + \dots$$

This is the sum over Regge poles

Physical interpretation of Regge poles



Complete theoretical treatment

Complete theory is more complex because it takes into account other different phenomena, in particular,

- Truncated mirror geometry
- Non zero diffuseness of the Fermi potential (position of Regge poles is very sensitive to the form of the upper part of the potential)

Some conclusions and perspectives

- This "whispering gallery" problem is closely related to other physical phenomena (rainbows, glorias,…)
- It could be particularly interesting for neutrons of lower energies
- Experimental picture is very reach (double distributions – angle and velocity)
- This system is potentially an excellent tool to study interaction of the neutron with the surface